Wigner Distribution, Partial Coherence, and Phase-Space Optics

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Abstract: The Wigner distribution is presented as a perfect means to treat partially coherent optical signals and their propagation through first-order optical systems from a radiometric and phase-space optical perspective.

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Summary

In 1932, Wigner introduced a distribution function in mechanics that permitted a description of mechanical phenomena in a phase space [1]. Such a Wigner distribution was introduced in optics by Dolin and Walther in the sixties [2, 3], to relate partial coherence to radiometry. A few years later, the Wigner distribution was introduced in optics again, especially in the area of Fourier optics [4], and since then, a great number of applications of the Wigner distribution have been reported.

In this tutorial paper, we will review the Wigner distribution and some of its applications to optical problems, especially with respect to partial coherence and first-order optical systems [5]. The paper is roughly an extension to two dimensions of a previous review paper on the application of the Wigner distribution to partially coherent light [6], with additional material taken from some more recent papers on the twist of partially coherent Gaussian light beams and on second- and higher-order moments of the Wigner distribution. Some parts of this paper have already been presented before [7–9]; they have also been used as the basis for a lecture on "Representation of signals in a combined domain: Bilinear signal dependence" at the Winter College on Quantum and Classical Aspects of Information Optics, The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy, January 2006, see [10].

While the mechanical phase space is connected to classical mechanics, where the movement of particles is studied, the phase space in optics is connected to geometrical optics, where the propagation of optical rays is considered. And where the position and momentum of a particle are the two important quantities in mechanics, in optics we are interested in the position and the direction of an optical ray. We will see that the Wigner distribution represents an optical field in terms of a ray picture, and that this representation is independent of whether the light is partially coherent or completely coherent.

We will observe that a description by means of a Wigner distribution is in particular useful when the optical signals and systems can be described by quadratic-phase functions, i.e., when we are in the realm of first-order optics: spherical waves, thin lenses, sections of free space in the paraxial approximation, etc. Although formulated in Fourier-optical terms, the Wigner distribution will form a link to such diverse fields as geometrical optics, ray optics, matrix optics, and radiometry.

The tutorial is roughly divided in seven parts. In parts 1 through 6, we will mainly deal with optical signals and systems. We treat the description of completely coherent and partially coherent light fields in part 1. The Wigner distribution is introduced in part 2 and elucidated with some optical examples. Properties of the Wigner distribution are considered in part 3. In part 4 we restrict ourselves to the one-dimensional case and observe the strong connection of the Wigner distribution to the fractional Fourier transformation and rotations in phase space. The propagation of the Wigner distribution through Luneburg's first-order optical systems is the topic of part 5, while the propagation of its moments is discussed in part 6. The final part 7 is devoted to the broad class of bilinear signal representations known as the Cohen class [11, 12], of which the Wigner distribution is an important representative.

The reader who is interested in phase-space optics, or time-frequency signal analysis, has a number of very useful references available. We mention in particular the easily accessible paper by Franz Hlawatsch and G. Fay Boudreaux Bartels [13], written from a signal-processing view point. The same holds for the book by Boualem Boashash [14]. Three other references are directly directed to optics and phase space [10, 15, 16].

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