Synthesis of an arbitrary ABCD-system, based on the modified Iwasawa decomposition

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Abstract: A lossless first-order optical system (or **ABCD** system) is described by its symplectic ray transformation matrix [**A** , **B** ; **C** , **D**]. After normalizing this matrix,

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} = \begin{bmatrix} \mathbf{w} & \mathbf{0} \\ \mathbf{0} & \mathbf{w}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{c} & \mathbf{d} \end{bmatrix} \begin{bmatrix} \mathbf{w}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{w} \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} w_x & 0 \\ 0 & w_y \end{bmatrix}, \quad w_x\,, w_y > 0\,,$$

the normalized **abcd** matrix (with ten degrees of freedom in the two-dimensional case) can be decomposed into the so-called modified Iwasawa decomposition [1, 2] $L(\mathbf{g})\, M(\mathbf{s})\, U(\mathbf{x}+\mathrm{i}\mathbf{y})$, where the first matrix $L(\mathbf{g}) = [\mathbf{I}, \mathbf{0}\,; -\mathbf{g}, \mathbf{I}]$ is described by the symmetric matrix $\mathbf{g} = -\,(\mathbf{ca}^{\mathrm{t}} + \mathbf{db}^{\mathrm{t}})\,(\mathbf{aa}^{\mathrm{t}} + \mathbf{bb}^{\mathrm{t}})^{-1}$, the matrix $M(\mathbf{s}) = [\mathbf{s}\,, \mathbf{0}\,; \mathbf{0}\,, \mathbf{s}^{-1}]$ by the positive-definite symmetric matrix $\mathbf{s} = (\mathbf{aa}^{\mathrm{t}} + \mathbf{bb}^{\mathrm{t}})^{1/2}$, and the final matrix $U(\mathbf{x}+\mathrm{i}\mathbf{y}) = [\mathbf{x}\,, \mathbf{y}\,; -\mathbf{y}\,, \mathbf{x}]$ by the unitary matrix $\mathbf{x} + \mathrm{i}\mathbf{y} = (\mathbf{aa}^{\mathrm{t}} + \mathbf{bb}^{\mathrm{t}})^{-1/2}\,(\mathbf{a} + \mathrm{i}\mathbf{b})$.

$L(\mathbf{g})$ corresponds to an anamorphic lens (i.e., two crossed cylindrical lenses, oriented at the appropriate angle; hence three degrees of freedom). $M(\mathbf{s})$ corresponds to a separable magnifier (using two crossed cylindrical lenses with proper orientation; three degrees of freedom). $U(\mathbf{x}+\mathrm{i}\mathbf{y})$ corresponds to a so-called orthosymplectic system (because its symplectic ray transformation matrix is also orthogonal) and can be realized, for instance, as a separable fractional Fourier transformer $F(\gamma_x, \gamma_y)$ (with different fractional angles γ_x and γ_y in the two perpendicular directions; two degrees of freedom) embedded in between two rotators $R(\alpha)$ and $R(\beta)$ (with different rotation angles α and β; two degrees of freedom): $U = R(\beta)\, F(\gamma_x, \gamma_y)\, R(\alpha)$.

The Iwasawa decomposition yields normalized subsystems. To actually synthesize the different subsystems in reality, we have to re-introduce the normalizing matrix **w** again; this is rather straightforward, but leads to scale adapters around the fractional Fourier transformer. The normalization parameters w_x and w_y can then be chosen such that the number of optical elements is reduced. The resulting synthesis is a cascade of lenses and sections of free space, where the number of cylindrical lenses equals ten: two lenses for the anamorphic lens (which at the same time take care of the phase corrections for the separable magnifier); two lenses to realize the separable magnifier; two lenses for the separable fractional Fourier transformer; and two lenses for each of the scale adapters that embed the fractional Fourier transformer.

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